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Controlled movement and suppression of spiral waves in excitable media

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Here we propose mechanisms for controlling the movement and suppression of spiral waves in discrete excitable media. We show that the controlled drift and subsequent annihilation of a spiral wave can be achieved through the combination of two factors: the introduction of small spatial inhomogeneities in the medium and the interaction of the wave with the boundaries of the medium. The inhomogeneities can be introduced in the spatial distribution of the relaxation parameters or in the coupling coefficient of the slow variables of the partial cells making up the medium. [S1063-651X(98)00912-X]

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The appearance of spiral waves in excitable and oscillatory media is often an undesirable effect, leading to unpredictable consequences for many applications [1]. For instance, a single spiral wave in cardiac tissue has been identified as a likely cause of monomorphic ventricular tachycardia [2,3]. The spontaneous breakup of a spiral wave into several waves and their subsequent multiplication can lead to chaotic behavior [4]. Such dynamics have been implicated as a mechanism for the onset of ventricular fibrillation [5], which is one of the leading causes of death in industrialized countries. Accordingly, there is a need to develop effective methods to control and annihilate spiral waves. We should also note that spiral waves are considered to be interesting objects of observation and control in other biological systems [6] and many chemical [7] and physical systems [8].

Several methods for controlling and/or suppressing [9] single spiral waves have been proposed. One of the more effective schemes involves the application of an external driving signal (e.g., field) to the medium sustaining the spiral wave [10]. For example, a periodic, small-amplitude, uniform electric field can be used to initialize directed drift of spiral waves in excitable media. It has also been shown experimentally and computationally that spiral waves can be stabilized and/or destroyed via the application of global feedback [11]. In Ref. [12], it has been shown that the presence of parameter gradients can lead to drift and subsequent annihilation of a spiral wave at the boundary. It has also been demonstrated that geometry and the size of inhomogeneities can significantly influence wave dynamics in excitable media [13,14].

In this paper, we present a method that utilizes small perturbations to control and annihilate spiral waves in excitable media. With this method, we induce directed movement of a spiral wave toward a boundary *by introducing localized spatial inhomogeneities between the boundary and the core of the spiral wave* (in order to amplify the influence of the boundary [15]).

Here we consider a discrete model of an excitable medium with nearest-neighbor coupling of partial elements (see, for example, Ref. [16]):

$$\dot{u}_{i,j} = \varepsilon^{-1} f(u_{i,j}, v_{i,j})$$
$$\qquad + D_u(u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1} - 4u_{i,j}),$$
$$\dot{v}_{i,j} = \varepsilon g(u_{i,j}, v_{i,j}) \qquad\qquad\qquad (1)$$
$$\qquad + D_v(v_{i-1,j} + v_{i+1,j} + v_{i,j-1} + v_{i,j+1} - 4v_{i,j}),$$

where $i = 1, \ldots, M$ and $j = 1, \ldots, N$. The functions $f(u_{i,j}, v_{i,j})$ and $g(u_{i,j}, v_{i,j})$ have the forms

$$f(u_{i,j}, v_{i,j}) = u_{i,j} - u_{i,j}^3/3 - v_{i,j},$$
$$g(u_{i,j}, v_{i,j}) = u_{i,j} - \gamma v_{i,j} + \beta. \qquad\qquad (2)$$

Variables $u_{i,j}$ and $v_{i,j}$ in Eq. (1) are fast and slow variables, respectively, because we only consider small values for the relaxation parameter ε, which controls the spatiotemporal scale separation. Parameter β defines the asymmetry between the excitation and recovery for each element, and parameter $\gamma > 0$ characterizes the dissipation of the slow variable $v_{i,j}$. D_u and D_v are the coupling coefficients for the fast and slow variables, respectively. This system could be taken as a simple model for cardiac tissue, which consists of separate excitable cells coupled through gap junctions. (In the case of cardiac tissue, the slow variable $v_{i,j}$ does not diffuse, i.e., $D_v = 0$.)
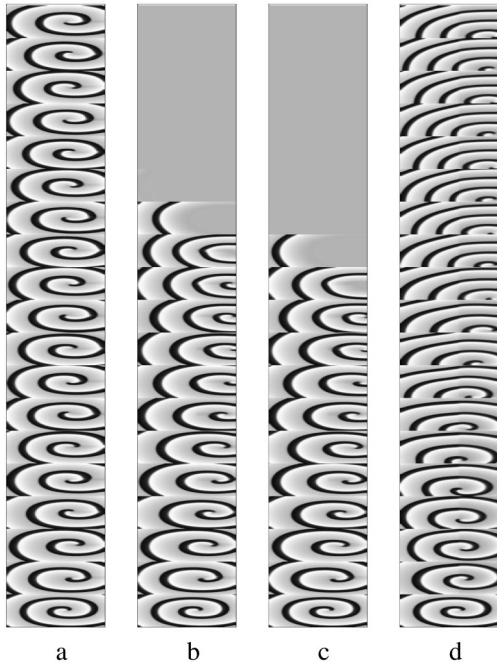
FIG. 1. Snapshots of the spatial distribution of the variable $u_{i,j}$ at different times (where i corresponds to the horizontal axis and j corresponds to the vertical axis). Note that time is increasing along the vertical axis [from 0 to 360 in (a), (b), and (c), and from 0 to 900 in (d)]. Parameters: $N=M=100, \varepsilon_1=0.3, \varepsilon_2=0.2, i_1=51, i_2=100$. (a) The tip of the original spiral wave does not drift. Parameters: $j_1=48, j_2=55$. (b),(c) The spiral wave drifts to the boundary and then gets annihilated. Parameters: (b) $j_1=47, j_2=56$, and (c) $j_1=46, j_2=57$. (d) The spiral wave core gets pinned at the boundary. Parameters: $j_1=3, j_2=98$.

At the border Γ of the medium, we consider free boundary conditions:

$$u_{0,j}=u_{1,j}, \quad u_{M+1,j}=u_{M,j}, \quad u_{i,0}=u_{i,1}, \quad u_{i,N+1}=u_{i,N},$$
$$\text{(3)}$$
$$v_{0,j}=v_{1,j}, \quad v_{M+1,j}=v_{M,j}, \quad v_{i,0}=v_{i,1}, \quad v_{i,N+1}=v_{i,N}.$$

The partial-cell model [in Eq. (1) $D_u=D_v=0$] has a steady state at the intersection of the null clines $f(u_{i,j},v_{i,j})=0$ and $g(u_{i,j},v_{i,j})=0$. The coordinates, (u_s,v_s), and stability of the steady state are defined by the values of parameters β and γ. If a steady state of a partial cell lies in the interval $u<-1$ or the interval $u>1$, then it is stable. We consider the case when $u_s<-1$.

It can be shown that at $\gamma=0.5, \beta\geqslant 2/3$, the spatially homogeneous state with coordinates $u_{i,j}=u_s$ and $v_{i,j}=v_s$ is linearly stable in the medium. However, the spatially homogeneous state may be unstable to large perturbations, which can lead to the appearance of inhomogeneous motion. In particular, depending upon the perturbation, it is possible to obtain either a single spiral wave, an ensemble of interacting spiral waves, or spiral spatiotemporal chaos. In this study, we consider *a single spiral wave* (presented in the lower snapshots in all columns in Figs. 1 and 2) formed by particular initial conditions in the discrete excitable medium (1), for a 100×100 lattice of partial cells with parameter values of $\varepsilon=0.2, \; \beta=0.7, \; \gamma=0.5, \; D_u=2.0,$ and $D_v=0$ [17].
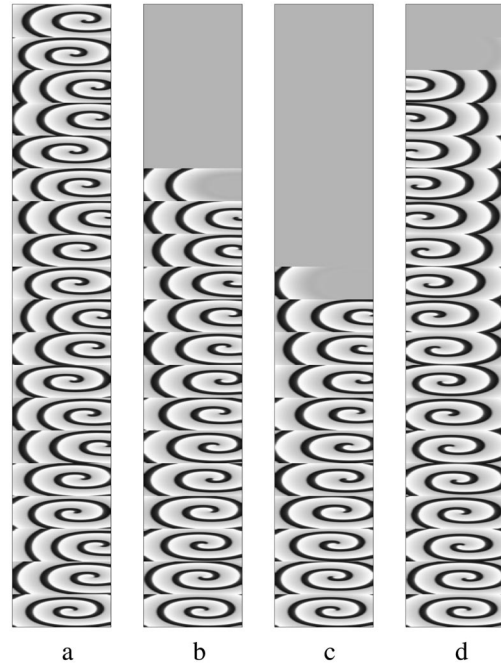

FIG. 2. Snapshots of the spatial distribution of the variable $u_{i,j}$ at different times (where i corresponds to the horizontal axis and j corresponds to the vertical axis). Note that time is increasing along the vertical axis [from 0 to 7200 in (a) and from 0 to 720 in (b), (c), and (d)]. Parameters: $N=M=100, \varepsilon=0.2, D_u=2, j_1=51, j_2=52$. (a) The interaction of the spiral wave with the boundary does not lead to the annihilation of the spiral; instead, it leads to its reflection from the boundary, which is repeated in this case at the opposite boundary, resulting in a crisscrossing pattern of movement. Parameters: $i_1=61, i_2=100, D_v^*=0.3$. (b), (c). The spiral wave drifts to the right boundary and then gets annihilated. Parameters: (b) $i_1=61, i_2=100, D_v^*=0.6$, (c) $i_1=61, i_2=100, D_v^*=1.4$. (d) The spiral wave drifts to the left boundary and then gets annihilated. Parameters: $i_1=1, i_2=60, D_v^*=0.9$.

The *critical size* of a spiral wave, denoted as Ω_{cr}, is the minimum size of the medium that can sustain a spiral wave. Numerical simulations showed that Ω_{cr} for the system we considered (as defined by the above parameter values) was a lattice of 12×12 partial cells. The region of reduced amplitude in the center of a spiral is called the *core* and is denoted as Ω_{core}. Numerical simulations showed that the core of the single spiral wave arising in the system we considered (with $\varepsilon=0.2$) was approximately inside a circle of radius four elements centered at the point $(i=55, j=51)$. We also found that if ε is increased to 0.35 ± 0.01 (for square lattices ranging from 12×12 to 100×100 partial cells), then the spiral wave becomes unstable and the system (1) migrates to the homogeneous steady state.

To induce controlled movement of the (unperturbed) spiral wave toward the media boundary, we introduce weakly inhomogeneous domains in the medium. We consider two types of inhomogeneity: (1) in the spatial distribution of the relaxation parameter ε, and (2) in the coupling coefficient (D_v) of the slow variable. Inhomogeneities in the relaxation parameter ε correspond to the conventional situation for many excitable systems, such as cardiac tissue and chemical reactions. In systems where the slow variable corresponds to the concentration of interacting substances (e.g., as in chemi-

cal systems, ecological systems, and systems describing burning) or organisms (e.g., as in colonies of bacteria), the diffusion coefficient of the slow variable can naturally be inhomogeneous. It can also be changed inhomogeneously via the introduction of certain external actions, such as localized temperature changes, illumination, and oxygen inhibition, i.e., any action that leads to a localized change in the concentration of substances corresponding to the slow variable.

In this study we first consider the effects of introducing inhomogeneities in the spatial distribution of the relaxation parameter ε. In our numerical experiments, the value of ε was spatially distributed according to the following relationship:

$$\varepsilon = \begin{cases} \varepsilon_1, & (i,j) \in \Omega_\varepsilon \\ \varepsilon_2, & (i,j) \notin \Omega_\varepsilon \end{cases} \qquad (4)$$

where $\Omega_\varepsilon = \{(i,j) | i_1 \le i \le i_2 ; j_1 \le j \le j_2\}$ is the domain of inhomogeneity, $i_{1,2}$ and $j_{1,2}$ define the borders of the domain, and ε_1 and ε_2 are the values of the relaxation parameter inside and outside of the domain, respectively.

We performed numerical experiments for different values of parameters i_1, i_2, j_1, j_2 and $\varepsilon_1, \varepsilon_2$. Figure 1 shows the calculated results for $i_1 = 51$, $i_2 = 100$, $\varepsilon_1 = 0.3$, and $\varepsilon_2 = 0.2$ for different values of j_1, j_2. Shown are sequential snapshots of the time evolution of the resulting spiral wave. The transition from the unperturbed spiral state [Fig. 1(a)] to directed movement takes place at $j_1 = 47$ and $j_2 = 56$, i.e., when the $(j_2 - j_1)$ width of the inhomogeneous strip practically coincides with the size of Ω_{cr} [18]. In Figs. 1(b) and 1(c), one can see directed drift of the spiral wave to the boundary and its subsequent annihilation. The influence of the boundary also manifests itself in Fig. 1(a) as a weak change in the shape of the spiral wave, i.e., the part of the spiral wave located inside the region Ω_ε is ''attracted'' to the boundary. We found that the drift velocity of the spiral wave increases with increasing size of the inhomogeneous region Ω_ε. We also found that the direction of the spiral-wave movement depends on the location and orientation of the spiral wave before the introduction of the inhomogeneity. In Fig. 1(d), it can be seen that the spiral-boundary interaction can also serve to anchor the spiral wave to the boundary. This finding is consistent with the results of Ref. [15].

To obtain directed movement and suppression of a spiral wave, as described above, we found that the domain of inhomogeneity must (1) have a width that is larger than or close to the critical size of the spiral wave, i.e., $\Omega_{cr} \subseteq \Omega_\varepsilon$, (2) extend to the one of the boundaries, i.e., $\Omega_\varepsilon \cup \Gamma \neq \varnothing$, and (3) almost cover the core of the spiral wave, i.e., $\Omega_{core} \cup \Omega_\varepsilon \neq \varnothing$. Note that the core of the spiral wave remains within the inhomogeneous domain during the controlled-movement process.

We now consider the effects of introducing an inhomogeneous domain in the coupling coefficient of the slow variables $v_{i,j}$ of the partial cells making up the medium. The associated diffusion term [$D_v \neq 0$ in Eq. (1)] can be taken to be the result of the presence of a short-acting field that establishes a new type of coupling between the partial cells and media boundary within the inhomogeneous domain. If one considers the core of the spiral wave to be a particle, then the drift of the spiral wave can be represented as the movement of that particle in the short-acting field, with the source of the field lying in the vicinity of the spiral-wave core. The field, after reflection from the boundary, influences the particle, causing it to drift [19]. Note that the interaction of a spiral wave with a planar, no-flux boundary is equivalent to the interaction of two oppositely charged spiral waves [20].

The coupling parameter D_v is spatially distributed according to the following relationship:

$$D_v = \begin{cases} D_v^*, & (i,j) \in \Omega_{D_v} \\ 0, & (i,j) \notin \Omega_{D_v} \end{cases} \qquad (5)$$

where $\Omega_{D_v} = \{(i,j) | i_1 \le i \le i_2 ; j_1 \le j \le j_2\}$ is the domain of inhomogeneity, and $i_{1,2}$ and $j_{1,2}$ define the borders of the domain.

We performed numerical experiments for different values of parameters i_1, i_2, j_1, j_2, and ε. Figure 2 shows the calculated results for $j_1 = 51$, $j_2 = 52$, $D_u = 2$, and $\varepsilon = 0.2$ for different values of i_1, i_2. Shown are sequential snapshots of the time evolution of the resulting spiral wave. We found that for successful spiral-wave suppression, one of two conditions must be met: (1) the domain Ω_{D_v} must be sufficiently narrow (i.e., the width of the strip must be less than the critical size of the spiral wave, but greater than the width of one partial cell) and the domain of inhomogeneity must intersect the region of the core, i.e., $\Omega_{core} \cup \Omega_{D_v} \neq \varnothing$, or (2) the domain Ω_{D_v} can be relatively large (i.e., $\Omega_{core} \subset \Omega_{D_v}$), but in such a case, the drift of the spiral wave can occur only for a small region of D_v^*. This means that for most of the values of D_v^*, the size of the inhomogeneous (perturbed) region Ω_{D_v} must be large enough to sustain the spiral wave. In both cases, the necessary condition for the annihilation of the spiral wave is that the inhomogeneous domain must extend to one of the boundaries, i.e., $\Omega_{D_v} \cup \Gamma \neq \varnothing$. Successful spiral-wave suppression is presented in Figs. 2(b)–2(d). Note that the deformation of the spiral wave, resulting from the introduced inhomogeneity, is enhanced if the value of the slow-variable coupling coefficient is increased.

We also observed ''competition'' effects between boundaries. Specifically, we found that if the inhomogeneous domain Ω_{D_v} connects two opposite boundaries, then the spiral wave moves to the nearest boundary. We also found that at some parameter values, the interaction of the spiral wave with the no-flux boundary does not lead to the annihilation of the spiral wave; instead, it leads to the reflection of the spiral wave from the boundary, followed by movement of the spiral wave toward the opposite boundary. This process can be repeated, leading to a crisscrossing pattern of movement [Fig. 2(a)]. This effect is consistent with our proposed particle-field mechanism for spiral-boundary interaction.

This work clearly shows that the movement of a spiral wave in excitable media can be controlled: (1) by changing the level of relaxation in part of the medium that connects the core of the spiral wave with the media boundary, or (2) by changing the level of coupling between the slow variables of partial cells located inside a region of the medium. These novel methods are based on the introduction of inhomogeneities and rely on the amplification of the interaction of the spiral wave with the boundary. Each method results in the

directed drift of the spiral wave to the boundary, leading to the annihilation of the spiral wave. These methods could be implemented experimentally in chemical reactions, for example, using localized illumination [14].

These methods were successfully used in this study to suppress spiral waves in active excitable media (i.e., $\beta = 0.2$). Because these methods are based on common properties of spiral waves, they can also be applied to different media and used over a wide range of parameter values. How-ever, inappropriate parameter values could lead to unpredictable effects, such as the appearance of spiral-wave chaos.

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[17] We also performed numerical experiments with differently sized lattices and conducted numerical analyses on a continuous-in-space model corresponding to our discrete model with fixed medium sizes and different grid sizes of 100×100 and 200×200, respectively. These calculations yielded results that were qualitatively similar to those reported in the main text for the 100×100 lattice.

[18] It should be noted that the critical size has been defined for the region with no-flux boundary conditions, whereas in our numerical simulations the partial cells in the region of inhomogeneity Ω_ε are coupled with cells outside of the region Ω_ε.

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